
Unit 8 (Part B). Simultaneous equation models

Application: Consider the following demand-supply model of the American food market for 1921-1940:

$$\left\{ \begin{array}{ll} QD_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t & \text{Demand function} \\ QS_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t & \text{Supply function} \end{array} \right.$$

where

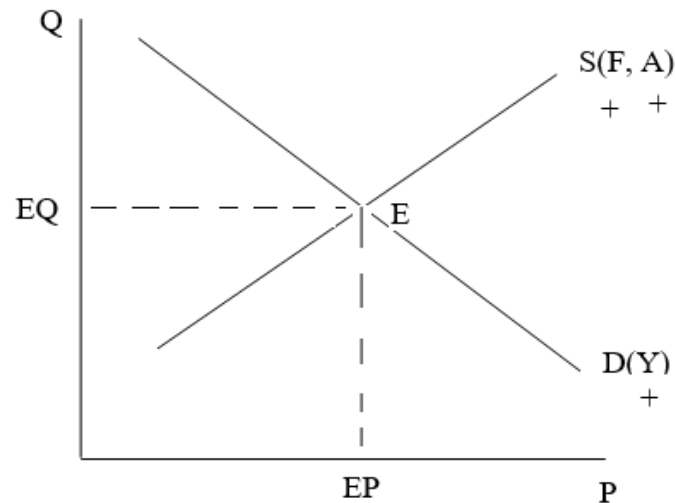
- QD_t = real food consumed per capita (quantity demanded)
- QS_t = real food supplied per capita (quantity supplied)
- P_t = relative food prices,
- Y_t = real disposable income per capita,
- F_t = preceding year's real income of farmers for their products, and
- A_t = time in years. Note that A_t is a time trend (in years) and used as a proxy for technological advances in the production of food (over time).

Below is the model at **equilibrium** ($Q_t = QD_t = QS_t$), where Q_t = quantity of food demanded and supplied.

$$\left\{ \begin{array}{ll} Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t & \text{Demand function} \\ Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t & \text{Supply function} \end{array} \right.$$

Q1. What are the theoretically expected signs for the slope parameters in the demand function (α_2 and α_3) and for the slope parameters in the supply function (β_2 , β_3 , and β_4).

{	$Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t$	Demand function	Y_t is the shift variable in the demand function.
	$Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$	Supply function	F_t and A_t are the shift variables in the supply function.



- Theoretically expected signs are: $\alpha_2 < 0$, $\alpha_3 > 0$, $\beta_2 > 0$, $\beta_3 > 0$, and $\beta_4 > 0$.

Q2. Present the data in addition to the time plots of the variables.

Eviews instructions:

Smpl 1921 1940

Show Q P Y F A

Plot Q

Plot P

Plot Y

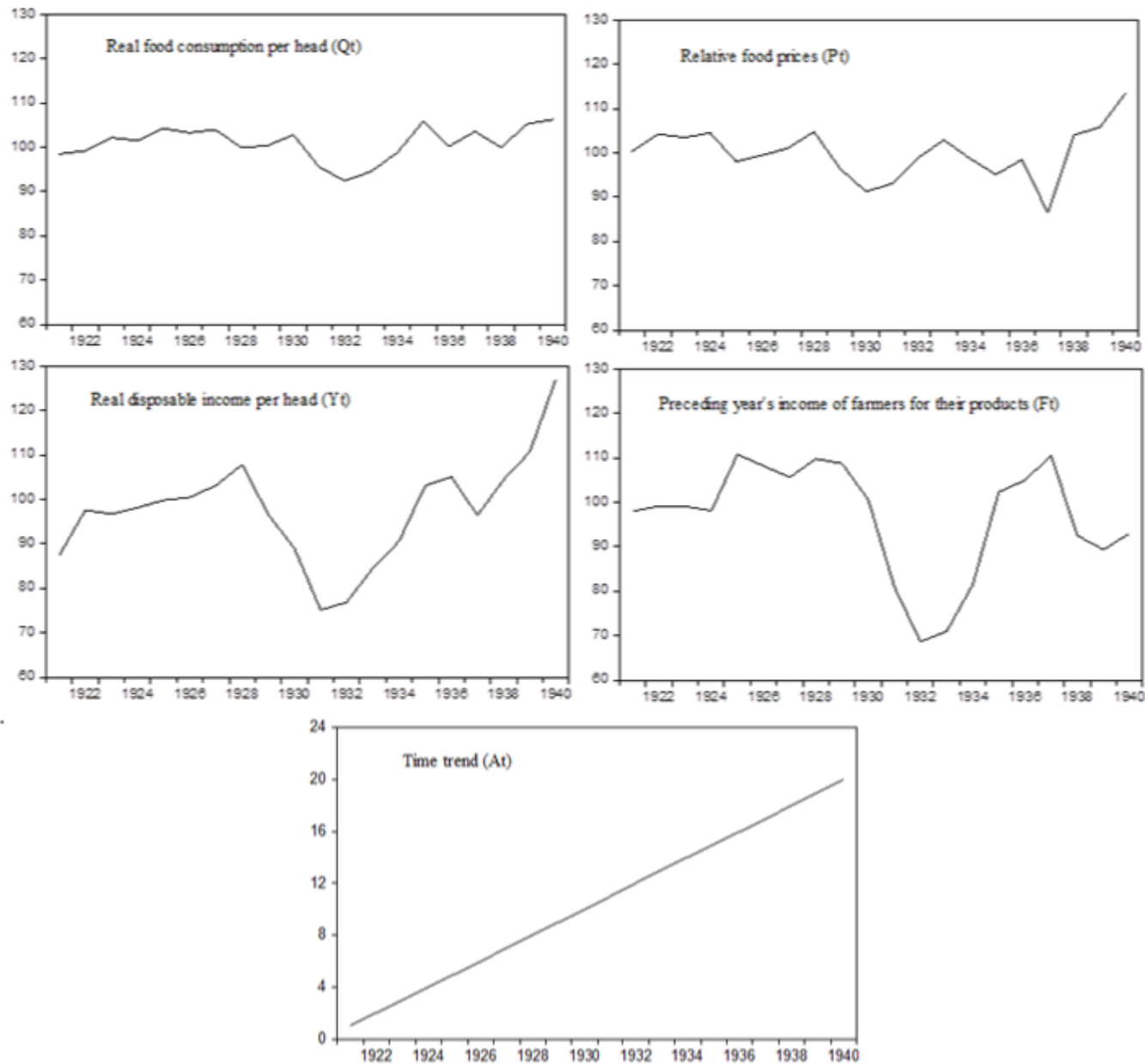
Plot F

Plot A

The data come from Kmenta, J. (1971). *Elements of econometrics* (No. 04; HB139, K5.)

Actual values of Q , P , Y , and F are in index points (with the average for 1935-39 = 100) as follows:

Year	Q_t	P_t	Y_t	F_t	A_t
1921	98.485	100.323	87.4	98	1
1922	99.187	104.264	97.6	99.1	2
1923	102.163	103.435	96.7	99.1	3
1924	101.504	104.506	98.2	98.1	4
1925	104.24	98.001	99.8	110.8	5
1926	103.243	99.456	100.5	108.2	6
1927	103.993	101.066	103.2	105.6	7
1928	99.9	104.763	107.8	109.8	8
1929	100.35	96.446	96.6	108.7	9
1930	102.82	91.228	88.9	100.6	10
1931	95.435	93.085	75.1	81	11
1932	92.424	98.801	76.9	68.6	12
1933	94.535	102.908	84.6	70.9	13
1934	98.757	98.756	90.6	81.4	14
1935	105.797	95.119	103.1	102.3	15
1936	100.225	98.451	105.1	105	16
1937	103.522	86.498	96.4	110.5	17
1938	99.929	104.016	104.4	92.5	18
1939	105.223	105.769	110.7	89.3	19
1940	106.232	113.49	127.1	93	20



Q3: Estimate the demand function using OLS. Do the slope estimates have the theoretically expected signs?

Views instruction:

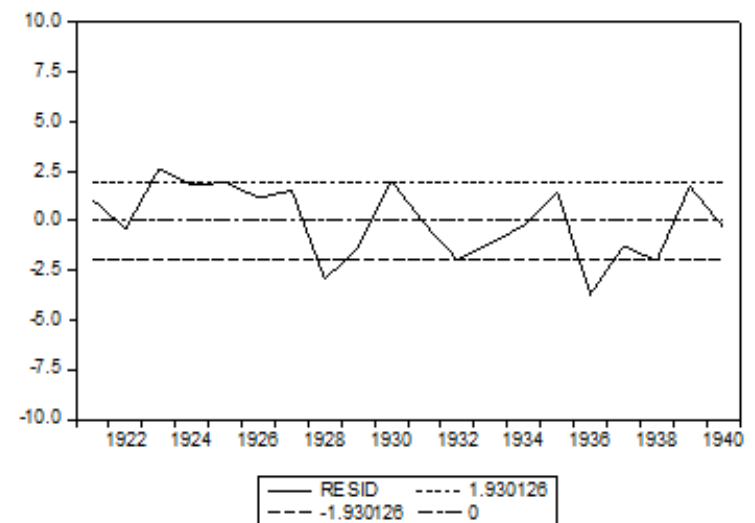
Smpl 1921 1940

LS Q c P Y

Plot Resid 0 1.930126 -1.930126

Dependent Variable: Q
Method: Least Squares
Sample: 1921 1940
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	99.89542	7.519360	13.28510	0.0000
P	-0.316299	0.090677	-3.488176	0.0028
Y	0.334636	0.045422	7.367287	0.0000
R-squared	0.763789	Mean dependent var	100.8982	
Adjusted R-squared	0.735999	S.D. dependent var	3.756497	
S.E. of regression	1.930126	Akaike info criterion	4.290529	
Sum squared resid	63.33160	Schwarz criterion	4.439889	
Log likelihood	-39.90529	Hannan-Quinn criter.	4.319686	
F-statistic	27.48473	Durbin-Watson stat	1.744204	
Prob(F-statistic)	0.000005			



- Yes, the slope estimates have the theoretically expected signs.

Q4: Estimate the supply function using OLS. Do the slope estimates have the theoretically expected signs?

Eviews instruction

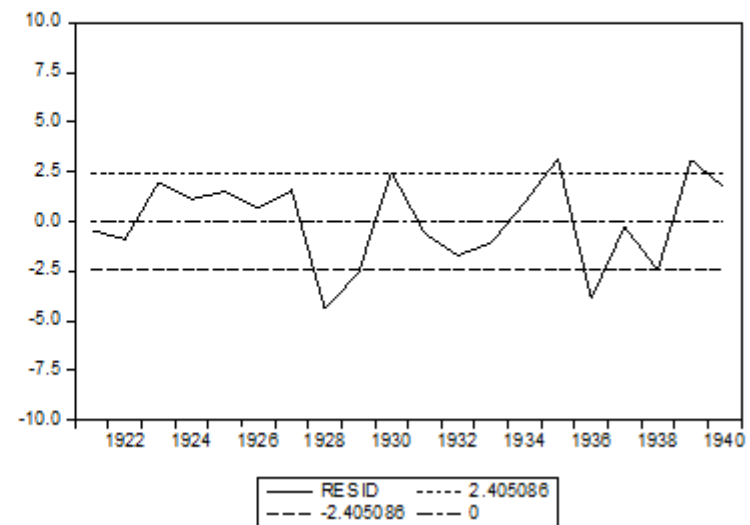
Smpl 1921 1940

LS Q c P F A

Plot Resid 0 2.405086 -2.405086

Dependent Variable: Q
Method: Least Squares
Sample: 1921 1940
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	58.27544	11.46291	5.083827	0.0001
P	0.160367	0.094884	1.690135	0.1104
F	0.248133	0.046188	5.372262	0.0001
A	0.248302	0.097518	2.546228	0.0216
R-squared	0.654807	Mean dependent var		100.8982
Adjusted R-squared	0.590084	S.D. dependent var		3.756497
S.E. of regression	2.405086	Akaike info criterion		4.769905
Sum squared resid	92.55102	Schwarz criterion		4.969051
Log likelihood	-43.69905	Hannan-Quinn criter.		4.808780
F-statistic	10.11698	Durbin-Watson stat		2.109732
Prob(F-statistic)	0.000560			



- Yes, the slope estimates have the theoretically expected signs.

Q5: Are the demand and supply functions behavioral or identities? Explain.

$$\left\{ \begin{array}{ll} Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t & \text{Demand function} \\ Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t & \text{Supply function} \end{array} \right.$$

- The demand function is **behavioral**, since it explains the behavior of households in demanding for food.

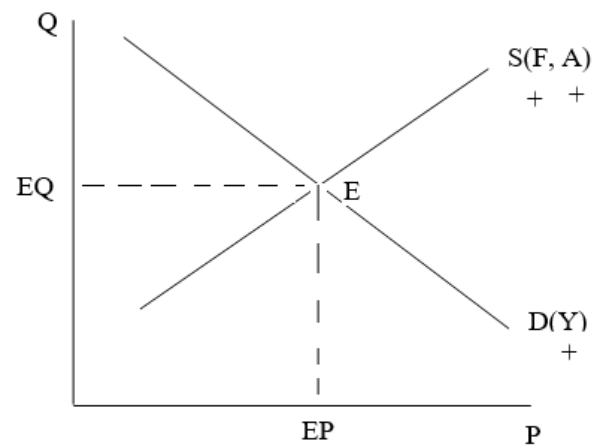
- The error term u_t measures the unexpected part of the dependent variable (quantity demanded).

- The supply function is **behavioral**, since it explains the behavior of farmers in supplying food.

- The error term v_t measures the unexpected part of the dependent variable (quantity supplied).

Q6: Which variables are endogenous and which are exogenous?

$$\begin{cases} Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t & \text{Demand function} \\ Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t & \text{Supply function} \end{cases}$$



- The **endogenous** variables are Q_t and P_t , since their values are determined within the model (i.e., the equilibrium values of the endogenous variables at point E. Note that there are 2 equations, which determine the equilibrium values of Q_t and P_t (as described by the theory of supply and demand).
- The remaining variables (Y_t , F_t , and A_t) are **exogenous**, since their values are determined outside the model.

Q7. Is the demand and supply model a structural or reduced-form model? Explain.

$$\begin{cases} Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t & \text{Demand function} \\ Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t & \text{Supply function} \end{cases}$$

- This demand and supply model is a structural model. This is because the demand function is a behavioral equation and includes the endogenous variable (P_t) on the right-hand side.
- In addition, the supply function is a structural equation. This is because the supply function is a behavioral equation and includes the endogenous variable (P_t) on the right-hand side.
- Very important: A reduced-form equation includes only exogenous variables on the right-hand side.

Q8. Write out the corresponding reduced-form model.

- **Structural model:**

$$Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t \quad \text{Demand function}$$

$$Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t \quad \text{Supply function}$$

- Note that $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$, and β_4 are called structural parameters.

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- The reduced-form model includes two reduced-form equations (one for each endogenous variable) as follows:

- **Reduced-form model:**

$$P_t = \pi_{11} + \pi_{12} Y_t + \pi_{13} F_t + \pi_{14} A_t + w_{1t} \quad \text{Reduced-form equation for } P_t$$

$$Q_t = \pi_{21} + \pi_{22} Y_t + \pi_{23} F_t + \pi_{24} A_t + w_{2t} \quad \text{Reduced-form equation for } Q_t$$

- Not that $\pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{24}$ are called reduced-form parameters.

- To derive the reduced-form equation for P_t , we set the right-hand terms in the demand function equal to the right-hand terms in the supply function as follows,

$$\alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$$

Solving for P_t yields the reduced-form equation,

$$P_t = \pi_{11} + \pi_{12} Y_t + \pi_{13} F_t + \pi_{14} A_t + w_{1t}$$

where

$$\pi_{11} = (\beta_1 - \alpha_1) / (\alpha_2 - \beta_2)$$

$$\pi_{12} = -\alpha_3 / (\alpha_2 - \beta_2)$$

$$\pi_{13} = \beta_3 / (\alpha_2 - \beta_2)$$

$$\pi_{14} = \beta_4 / (\alpha_2 - \beta_2)$$

$$w_{1t} = (v_t - u_t) / (\alpha_2 - \beta_2)$$

- Similarly, we can derive the reduced-form equation for Q_t ,

$$Q_t = \pi_{21} + \pi_{22} Y_t + \pi_{23} F_t + \pi_{24} A_t + w_{2t}$$

Q9: Show that classical assumption 2 is violated for the demand function. Discuss the theoretical and empirical consequences for the OLS estimators of α_1 , α_2 , and α_3 .

- As already stated, the demand function ($Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t$) is a **structural** equation, since it includes an endogenous variable (P_t) on the right-hand side.
- As such, following the procedure described in Part A, we can show that classical assumption 2 is violated, because the error term (u_t) is correlated with the endogenous variable (P_t).
- That is, $E(u_t P_t) \neq 0$
- For instance, $E[u_t(\pi_{11} + \pi_{12} Y_t + \pi_{13} F_t + \pi_{14} A_t + w_{1t})] = \sigma_u^2 \neq 0$
- **Theoretical consequences:** the OLS point estimators of α_1 , α_2 , and α_3 (in the demand function) are all biased.
- **Empirical consequences:** the OLS point estimates of α_1 , α_2 , and α_3 (in the demand function) are not usable for any purpose.

Q10: Show that classical assumption 2 is violated for the supply function. Discuss the theoretical and empirical consequences for the OLS estimators of β_1 , β_2 , β_3 , and β_4 .

- As already stated, the supply function ($Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$) is a **structural** equation, since it includes an endogenous variable (P_t) on the right-hand side.
- As such, following the procedure described in Part A, we can show that classical assumption 2 is violated, because the error term (u_t) is correlated with the endogenous variable (P_t).
- That is, $E(v_t \cdot P_t) \neq 0$
- For instance, $E[v_t \cdot (P_t = \pi_{21} + \pi_{22} Y_t + \pi_{23} F_t + \pi_{24} A_t + w_{2t})] = \sigma_v^2 \neq 0$
- **Theoretical consequences:** the OLS point estimators of β_1 , β_2 , β_3 , and β_4 (in the supply function) are all biased.
- **Empirical consequences:** the OLS point estimates of β_1 , β_2 , β_3 , and β_4 (in the supply function) are not usable for any purpose.

Q11: Is the demand function identified? Explain.

$$\left\{ \begin{array}{ll} Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t & \text{Demand function} \\ Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t & \text{Supply function} \end{array} \right.$$

- The answer is Yes.
- We say the demand function is identified, because there is at least one exogenous variable (other than Y_t) in the supply function. The exogenous variables in the supply function are F_t and A_t .
- **An important note:** We cannot use the TSLS method of estimation to estimate the demand function, if the demand function fails to be identified.
- We will explain this in Part C.

Q12: Is the supply function identified? Explain.

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$$Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t$$

Demand function

$$Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$$

Supply function
- The answer is Yes.
 - We say that the supply function is identified because there is at least one exogenous variable (other than F_t and A_t) in the demand function. The exogenous variable in the demand function is Y_t .
 - **An important note:** We cannot use the TSLS method of estimation to estimate the supply function, if the supply function fails to be identified.
 - We will explain this in Part C.

Q13: Use the two-stage least squares (TSLS) estimation method to estimate the demand functions. Present your TSLS SRF estimates.

Views instruction for the demand function:

Smpl 1921 1940

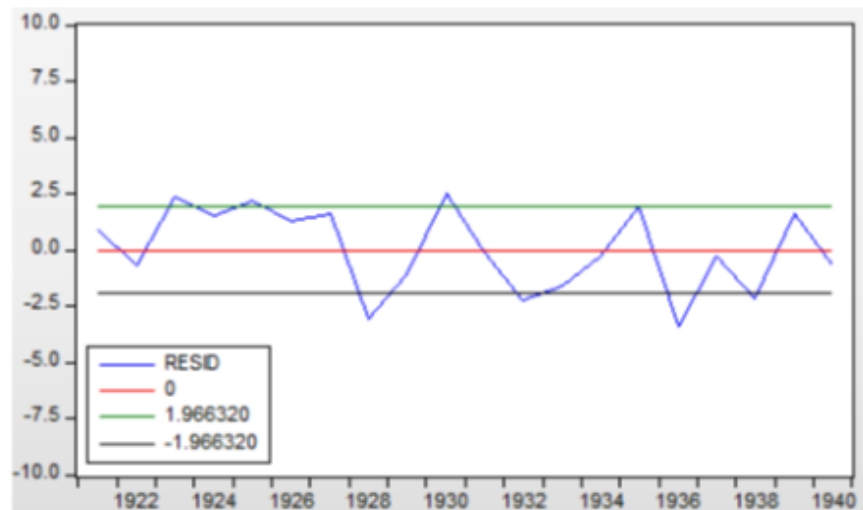
TSLS Q c P Y @ c Y F A

Plot Resid 0 1.966320 -1.966320

Dependent Variable: Q
Method: Two-Stage Least Squares

Sample: 1921 1940
Included observations: 20
Instrument specification: C Y F A

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	94.63330	7.920836	11.94739	0.0000
P	-0.243556	0.096484	-2.524313	0.0218
Y	0.313992	0.046944	6.688696	0.0000
R-squared	0.754847	Mean dependent var	100.8982	
Adjusted R-squared	0.726005	S.D. dependent var	3.756497	
S.E. of regression	1.966320	Sum squared resid	65.72904	
F-statistic	23.80653	Durbin-Watson stat	2.009222	
Prob(F-statistic)	0.000012	Second-Stage SSR	84.02241	
J-statistic	2.535644	Instrument rank	4	
Prob(J-statistic)	0.111301			



PRF (demand function):

$$Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t$$

TSLS SRF:

$$Q_t = \hat{\alpha}_1 + \hat{\alpha}_2 P_t + \hat{\alpha}_3 Y_t + e_t$$

TSLS SRF estimates:

$$Q_t = 94.63 - .244 P_t + .314 Y_t$$

(7.92) (.096) (.047) standard errors

(11.9) (2.52) (6.69) calculated t-values for the zero H_0

(.000) (.022) (.000) 2-tail test p -values for the calculated t-values

$R^2 = .755,$

Se = 1.9663,

DW = 2.01,

Cal. F (for H_0 : Pop. $R^2 = 0$) = 23.81 (.000)

Q14: Use the two-stage least squares (TSLS) estimation method to estimate the supply function. Present your TSLS SRF estimates of the supply function.

Views instruction for the supply function:

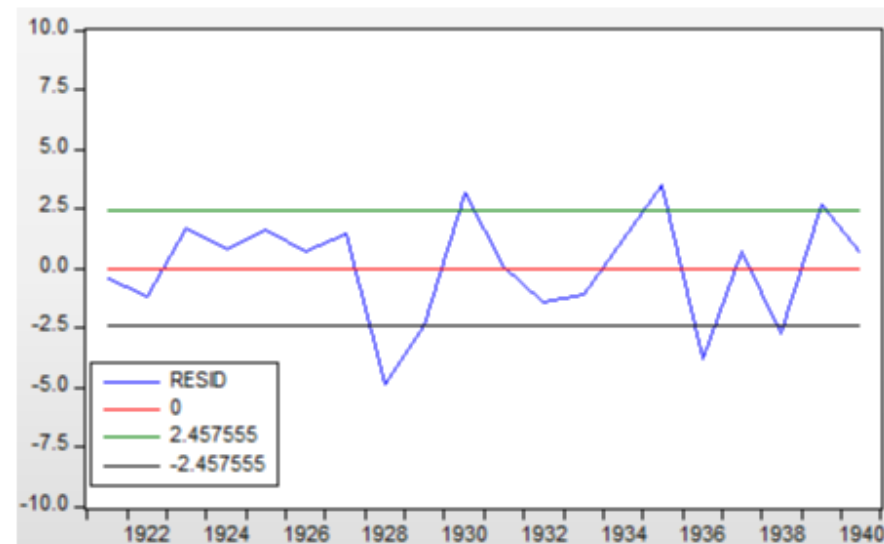
Smpl 1921 1940

TSLS Q c P F A @ c F A Y

Plot Resid 0 2.457555 -2.457555

Dependent Variable: Q
Method: Two-Stage Least Squares
Date: 10/02/21 Time: 08:28
Sample: 1921 1940
Included observations: 20
Instrument specification: C F A Y

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	49.53245	12.01052	4.124088	0.0008
P	0.240076	0.099934	2.402347	0.0288
F	0.255606	0.047250	5.409636	0.0001
A	0.252924	0.099655	2.537997	0.0219
R-squared	0.639582	Mean dependent var	100.8982	
Adjusted R-squared	0.572004	S.D. dependent var	3.756497	
S.E. of regression	2.457555	Sum squared resid	96.63320	
F-statistic	10.70139	Durbin-Watson stat	2.384646	
Prob(F-statistic)	0.000420	Second-Stage SSR	74.21856	
J-statistic	0.000000	Instrument rank	4	



PRF (supply function):

$$Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$$

TSLS SRF:

$$Q_t = \hat{\beta}_1 + \hat{\beta}_2 P_t + \hat{\beta}_3 F_t + \hat{\beta}_4 A_t + e_t$$

TSLS SRF estimates:

$$Q_t = 49.53 + .240 P_t + .256 F_t + .253 A_t$$

(12.0) (.099) (.047) (.099) standard errors

(4.12) (2.40) (5.41) (2.54) calculated t-values for the zero H_0

(.001) (.029) (.000) (.022) 2-tail test p -values

$R^2 = .640,$

Se = 2.458,

DW = 2.38,

Cal. F (for H_0 : Pop. $R^2 = 0$) = 10.70 (.0004)

Q15: Use the LM test to see if the demand function suffers from an autocorrelation problem (up to the 2nd-order).

- The demand function: $Q_t = a_1 + a_2 P_t + a_3 Y_t + u_t$

Eviews instructions: Smpl 1921 1940
TSLS Q c P Y @ c Y F A
Test(S,2)

- The LM test equation is: $e_t = a_0 + a_1 P_t + a_2 Y_t + \rho_1 e_{t-1} + \rho_2 e_{t-2} + \varepsilon_t$

The null and alternative hypotheses (for the overall test of autocorrelation up to the 2nd-order) are:

$$\left\{ \begin{array}{ll} H_0: \rho_1 = \rho_2 = 0 & \text{No autocorrelation problem up to the 2nd-order} \\ H_1: \text{At least one of } \rho\text{'s} \neq 0 & \text{Otherwise} \end{array} \right.$$

- From the Eviews output below, the calculated χ^2 -value is .0125 with a p -value of .9938 > .10. As such, we cannot reject $H_0: \rho_1 = \rho_2 = 0$ and, thus, there exists no autocorrelation problem up to the 2nd-order.
- This conclusion holds when we examine the individual test: $H_0: \rho_1 = 0$ and $H_0: \rho_2 = 0$.

- The LM test equation is:

$$e_t = a_0 + a_1 P_t + a_2 Y_t + \rho_1 e_{t-1} + \rho_2 e_{t-2} + \varepsilon_t$$

Dependent Variable: Q
Method: Two-Stage Least Squares
Sample: 1921 1940
Included observations: 20
Instrument specification: C Y F A

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	94.63330	7.920836	11.94739	0.0000
P	-0.243556	0.096484	-2.524313	0.0218
Y	0.313992	0.046944	6.688696	0.0000
R-squared	0.754847	Mean dependent var	100.8982	
Adjusted R-squared	0.726005	S.D. dependent var	3.756497	
S.E. of regression	1.966320	Sum squared resid	65.72904	
F-statistic	23.80653	Durbin-Watson stat	2.009222	
Prob(F-statistic)	0.000012	Second-Stage SSR	84.02241	
J-statistic	2.535644	Instrument rank	4	
Prob(J-statistic)	0.111301			

Breusch-Godfrey Serial Correlation LM Test

Obs*R-squared	0.012478	Prob. Chi-Square(2)	0.9938
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Test Equation:
Dependent Variable: RESID
Method: Two-Stage Least Squares

Sample: 1921 1940
Included observations: 20
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.026196	8.988184	-0.002914	0.9977
P	0.000472	0.107318	0.004394	0.9966
Y	-0.000220	0.050013	-0.004408	0.9965
RESID(-1)	-0.013663	0.272796	-0.050085	0.9607
RESID(-2)	-0.021436	0.268862	-0.079729	0.9375
R-squared	0.000624	Mean dependent var	2.71E-15	
Adjusted R-squared	-0.265876	S.D. dependent var	1.859952	
S.E. of regression	2.092654	Akaike info criterion	4.527061	
Sum squared resid	65.68803	Schwarz criterion	4.775995	
Log likelihood	-40.27061	Hannan-Quinn criter.	4.575656	
F-statistic	0.002341	Durbin-Watson stat	1.987918	
Prob(F-statistic)	0.999988			

- The null hypothesis for the overall test: $H_0: \rho_1 = \rho_2 = 0$
- The null hypotheses for the individual tests: $H_0: \rho_1 = 0$ and $H_0: \rho_2 = 0$.

Q16: Use the LM test to see if the supply function suffers from an autocorrelation problem (up to the 2nd-order).

- The supply function:

$$Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$$

Eviews instructions:

Smpl 1921 1940

TSLS Q c P F A @ c F A Y

Test(S,2)

- The LM test equation is:

$$e_t = a_0 + a_1 P_t + a_2 F_t + a_3 A_t + \rho_1 e_{t-1} + \rho_2 e_{t-2} + \varepsilon_t$$

The null and alternative hypotheses (for the overall test of autocorrelation up to the 2nd-order) are:

$$\left\{ \begin{array}{ll} H_0: \rho_1 = \rho_2 = 0 & \text{No autocorrelation problem up to the 2nd-order} \\ H_1: \text{At least one of } \rho\text{'s} \neq 0 & \text{Otherwise} \end{array} \right.$$

- From the Eviews output below, the calculated χ^2 -value is 2.2307 with a p -value of .3278 > .10. As such, we cannot reject $H_0: \rho_1 = \rho_2 = 0$ and, thus, there exists no autocorrelation problem up to the 2nd-order.
- This conclusion holds when we examine the individual test: $H_0: \rho_1 = 0$ and $H_0: \rho_2 = 0$.

- The LM test equation is:

$$e_t = a_0 + a_1 P_t + a_2 F_t + a_3 A_t + \rho_1 e_{t-1} + \rho_2 e_{t-2} + \varepsilon_t$$

Dependent Variable: Q
Method: Two-Stage Least Squares
Sample: 1921 1940
Included observations: 20
Instrument specification: C F A Y

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	49.53245	12.01052	4.124088	0.0008
P	0.240076	0.099934	2.402347	0.0288
F	0.255606	0.047250	5.409636	0.0001
A	0.252924	0.099655	2.537997	0.0219
R-squared	0.639582	Mean dependent var	100.8982	
Adjusted R-squared	0.572004	S.D. dependent var	3.756497	
S.E. of regression	2.457555	Sum squared resid	96.63320	
F-statistic	10.70139	Durbin-Watson stat	2.384646	
Prob(F-statistic)	0.000420	Second-Stage SSR	74.21856	
J-statistic	0.000000	Instrument rank	4	

Breusch-Godfrey Serial Correlation LM Test

Obs*R-squared	2.230747	Prob. Chi-Square(2)	0.3278
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Test Equation:
Dependent Variable: RESID
Method: Two-Stage Least Squares
Sample: 1921 1940
Included observations: 20
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.851821	12.55792	-0.147462	0.8849
P	0.013602	0.106455	0.127774	0.9001
F	0.006052	0.047922	0.126294	0.9013
A	-0.014239	0.101140	-0.140781	0.8900
RESID(-1)	-0.268580	0.270105	-0.994352	0.3369
RESID(-2)	-0.289816	0.277642	-1.043845	0.3142
R-squared	0.111537	Mean dependent var	8.60E-15	
Adjusted R-squared	-0.205771	S.D. dependent var	2.255207	
S.E. of regression	2.476388	Akaike info criterion	4.894804	
Sum squared resid	85.85498	Schwarz criterion	5.193524	
Log likelihood	-42.94804	Hannan-Quinn criter.	4.953118	
F-statistic	0.351511	Durbin-Watson stat	2.224200	
Prob(F-statistic)	0.872840			

- The null hypothesis for the overall test: $H_0: \rho_1 = \rho_2 = 0$
- The null hypotheses for the individual tests: $H_0: \rho_1 = 0$ and $H_0: \rho_2 = 0$.

Q17: Use the White test to check whether the demand function suffers from a heteroscedasticity problem.

- The demand function: $Q_t = a_1 + a_2 P_t + a_3 Y_t + u_t$

Eviews instructions: Smpl 1921 1940

TSLS Q c P Y @ c Y F A

Test(k)

- The White test equation is: $e_t^2 = \delta_0 + \delta_1 P_t^2 + \delta_2 Y_t^2 + \varepsilon_t$

The null and alternative hypotheses for the overall test of heteroscedasticity are:

$$\left\{ \begin{array}{ll} H_0: \delta_1 = \delta_2 = 0 & \text{No heteroscedasticity problem} \\ H_1: \text{At least one of } \delta\text{'s} \neq 0 & \text{Otherwise} \end{array} \right.$$

- From the Eviews output, the calculated F-value (for $H_0: \delta_1 = \delta_2 = 0$) is .1971 with a p -value of .8229 > .10
- As such, we cannot reject $H_0: \delta_1 = \delta_2 = 0$, meaning that there exists no heteroscedasticity problem.
- The same conclusion holds when we examine the individual tests ($H_0: \delta_1 = 0$ and $H_0: \delta_2 = 0$).

- The White test equation is: $e_t^2 = \delta_0 + \delta_1 P_t^2 + \delta_2 Y_t^2 + \varepsilon_t$

Dependent Variable: Q
Method: Two-Stage Least Squares
Sample: 1921 1940
Included observations: 20
Instrument specification: C Y F A

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	94.63330	7.920836	11.94739	0.0000
P	-0.243556	0.096484	-2.524313	0.0218
Y	0.313992	0.046944	6.688696	0.0000
R-squared	0.754847	Mean dependent var	100.8982	
Adjusted R-squared	0.726005	S.D. dependent var	3.756497	
S.E. of regression	1.966320	Sum squared resid	65.72904	
F-statistic	23.80653	Durbin-Watson stat	2.009222	
Prob(F-statistic)	0.000012	Second-Stage SSR	84.02241	
J-statistic	2.535644	Instrument rank	4	
Prob(J-statistic)	0.111301			

Heteroskedasticity Test: White

F-statistic	0.197127	Prob. F(2,17)	0.8229
Obs*R-squared	0.453315	Prob. Chi-Square(2)	0.7972
Scaled explained SS	0.147193	Prob. Chi-Square(2)	0.9290

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 10/19/21 Time: 13:33
Sample: 1921 1940
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.502171	6.639787	0.527452	0.6047
P^2	-0.000269	0.000820	-0.328339	0.7467
Y^2	0.000258	0.000413	0.624262	0.5407
R-squared	0.022666	Mean dependent var	3.286452	
Adjusted R-squared	-0.092315	S.D. dependent var	3.196723	
S.E. of regression	3.341018	Akaike info criterion	5.387910	
Sum squared resid	189.7609	Schwarz criterion	5.537269	
Log likelihood	-50.87910	Hannan-Quinn criter.	5.417066	
F-statistic	0.197127	Durbin-Watson stat	2.707400	
Prob(F-statistic)	0.822937			

- The null hypothesis for the overall test: $H_0: \delta_1 = \delta_2 = 0$
- The null hypotheses for the individual tests: $H_0: \delta_1 = 0$ and $H_0: \delta_2 = 0$.

Q18: Use the White test to check whether the supply function suffers from a heteroscedasticity problem.

- The supply function: $Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$

Eviews instructions: Smpl 1921 1940
TSLS Q c P F A @ c F A Y
Test(k)

- The White test equation is: $e_t^2 = \delta_0 + \delta_1 P_t^2 + \delta_2 F_t^2 + \delta_3 A_t^2 + \varepsilon_t$

The null and alternative hypotheses for the overall test of heteroscedasticity are:

$H_0: \delta_1 = \delta_2 = \delta_3 = 0$	No heteroscedasticity problem
$H_1: \text{At least one of } \delta\text{'s} \neq 0$	Otherwise

- As seen, the calculated F-value (for testing $H_0: \delta_1 = \delta_2 = \delta_3 = 0$) is 1.0704 with a p -value of .3894 > .10.
- As such, we cannot reject $H_0: \delta_1 = \delta_2 = \delta_3 = 0$, meaning that there exists no heteroscedasticity problem.
- The same conclusion holds when we examine the individual tests ($H_0: \delta_1 = 0$, $H_0: \delta_2 = 0$, and $H_0: \delta_3 = 0$).

- The White test equation is:

$$e_t^2 = \delta_0 + \delta_1 P_t^2 + \delta_2 F_t^2 + \delta_3 A_t^2 + \varepsilon_t$$

Dependent Variable: Q
Method: Two-Stage Least Squares
Sample: 1921 1940
Included observations: 20
Instrument specification: C F A Y

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	49.53245	12.01052	4.124088	0.0008
P	0.240076	0.099934	2.402347	0.0288
F	0.255606	0.047250	5.409636	0.0001
A	0.252924	0.099655	2.537997	0.0219
R-squared	0.639582	Mean dependent var	100.8982	
Adjusted R-squared	0.572004	S.D. dependent var	3.756497	
S.E. of regression	2.457555	Sum squared resid	96.63320	
F-statistic	10.70139	Durbin-Watson stat	2.384646	
Prob(F-statistic)	0.000420	Second-Stage SSR	74.21856	
J-statistic	0.000000	Instrument rank	4	

Heteroskedasticity Test: White

F-statistic	1.070396	Prob. F(3,16)	0.3894
Obs*R-squared	3.343038	Prob. Chi-Square(3)	0.3417
Scaled explained SS	1.670439	Prob. Chi-Square(3)	0.6435

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 10/19/21 Time: 13:38
Sample: 1921 1940
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-9.999325	14.84321	-0.673663	0.5101
P^2	0.000288	0.001229	0.234669	0.8174
F^2	0.001091	0.000641	1.701570	0.1082
A^2	0.011015	0.011520	0.956164	0.3532

R-squared	0.167152	Mean dependent var	4.831660
Adjusted R-squared	0.010993	S.D. dependent var	6.194471
S.E. of regression	6.160330	Akaike info criterion	6.650994
Sum squared resid	607.1946	Schwarz criterion	6.850141
Log likelihood	-62.50994	Hannan-Quinn criter.	6.689870
F-statistic	1.070396	Durbin-Watson stat	2.087992
Prob(F-statistic)	0.389438		

- The null hypothesis for the overall test: $H_0: \delta_1 = \delta_2 = \delta_3 = 0$
- The null hypotheses for the individual tests: $H_0: \delta_1 = 0$, $\delta_2 = 0$, and $H_0: \delta_3 = 0$.

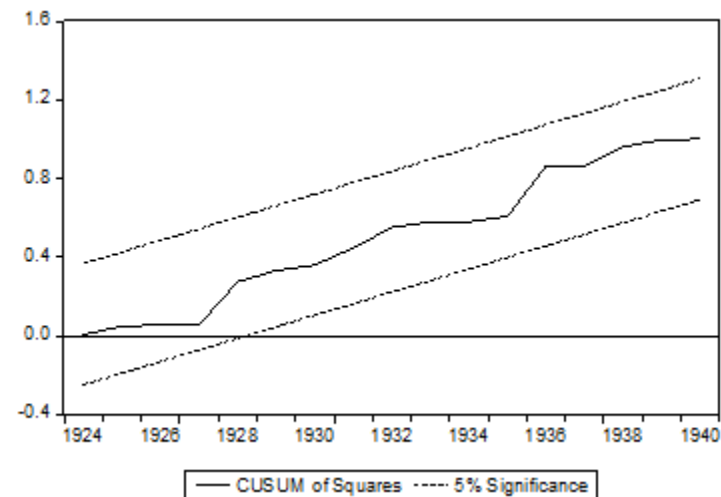
Q19: Use the cusum of squares test to check whether the demand function is stable in terms of parameter.

- The demand function:

$$Q_t = a_1 + a_2 P_t + a_3 Y_t + u_t$$

Eviews instruction:

Smpl 1921 1940
LS P c F A Y
Genr FP=P-Resid
LS Q c FP Y
Test(v)



The null and alternative hypotheses are:

H_0 : The demand function is stable in terms of parameters over the entire period

H_1 : Otherwise

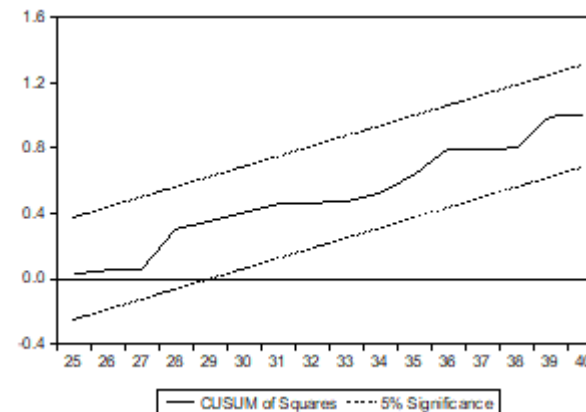
- From the cusum of squares time plot, we conclude that the demand function is stable in terms of parameters. This is because the calculated cusum of squares stays within the 5% critical boundaries.

Q20: Use the cusum of squares test to check whether the supply function is stable in terms of parameter.

- The supply function: $Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$

Eviews instruction:

Smpl 1921 1940
LS P c F A Y
Genr FP=P-Resid
LS Q c FP F A
Test(v)



The null and alternative hypotheses are:

- H_0 : The supply function is stable in terms of parameters over the entire period
- H_1 : Otherwise

- From the cusum of squares time plot (below), we conclude that the supply function is stable in terms of parameters. This is because the calculated cusum of squares stays within the 5% critical boundaries.

Q21: Does the demand function suffers from a high multicollinearity problem? Explain

- The demand function: $Q_t = a_1 + a_2 P_t + a_3 Y_t + u_t$

Eviews instruction: Smpl 1921 1940
COR P Y

Correlation		
	P	Y
P	1.000000	0.566549
Y	0.566549	1.000000

- From the correlation coefficient matrix, the correlation coefficient estimate among the independent variables (.576) is not nearly one.
- We thus conclude that the estimated demand function does not suffer from a high multicollinearity problem.

Q22: Does the supply function suffers from a high multicollinearity problem? Explain

- The supply function: $Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$

Eviews instruction: Smpl 1921 1940
COR P F A

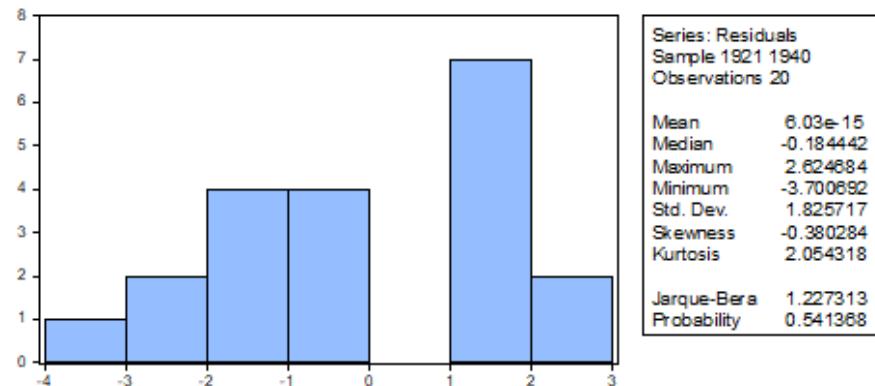
Correlation				
		P	F	A
P	P	1.000000	-0.184428	-0.000178
F	F	-0.184428	1.000000	-0.287041
A	A	-0.000178	-0.287041	1.000000

- From the correlation coefficient matrix, the absolute correlation coefficient estimates among the independent variables (varying from .000178 to .2870) are not nearly one.
- We thus conclude that the estimated supply function does not suffer from a high multicollinearity problem.

Q23: Is the normality assumption empirically valid for the demand function?

- The demand function: $Q_t = a_1 + a_2 P_t + a_3 Y_t + u_t$

Eviews instruction: Smpl 1921 1940
TSLS Q c P Y @ c P F A
Test(h)



The null and alternative hypotheses are:

H_0 : The error term u_t is normally distributed for all $t = 1, 2, \dots$

H_1 : Otherwise

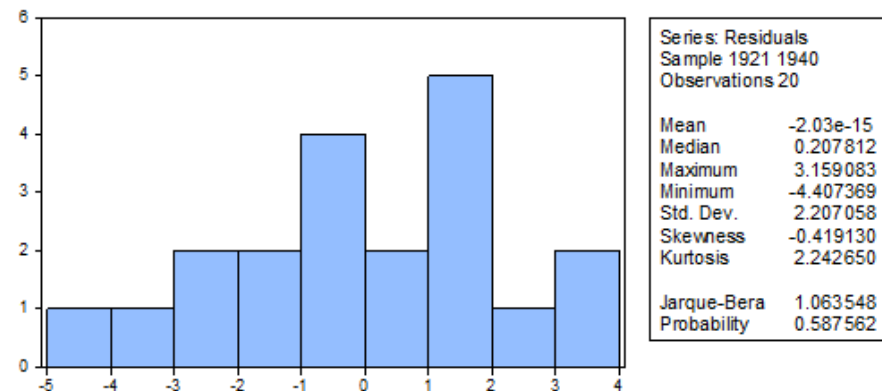
- From the Eviews output, the calculated JB test statistic (1.2273) is low with a p -value of .541 $>$.10. We thus fail to reject H_0 , concluding that the normality assumption is empirically valid.
- Note: The sample size is small ($n = 20$). As such, the Bera-Jarque test of normality is not reliable. For instance, the estimated coefficient of skewness (-.380) is not very close to 0 and the estimated coefficient of kurtosis (2.054) is not very close to 3 and, yet, we end up with a low calculated JB test statistic (1.227).

Q24: Is the normality assumption empirically valid for the supply function?

- The supply function: $Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$

Eviews instruction:

Smpl 1921 1940
TSLS Q c P F A @ c P F A Y
Test(h)



The null and alternative hypotheses are:

H_0 : The error term v_t is normally distributed for all $t = 1, 2, \dots$

H_1 : Otherwise

- From the Eviews output, the calculated JB test statistic (1.063) is low with a p -value of .587 > .10. We thus cannot reject H_0 , concluding that the normality assumption is empirically valid.
- Note: The sample size is small ($n = 20$), meaning that the Bera-Jarque test of normality is not reliable. For instance, the estimated coefficient of skewness (-.419) is not very close to 0 and the estimated coefficient of kurtosis (2.243) is not very close to 3 and, yet, we end up with a low calculated JB test statistic (1.063).

Q25: Are the signs of the parameters in the demand and supply functions theoretically correct?

PRF (demand function): $Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t$

TSLS SRF: $Q_t = \hat{\alpha}_1 + \hat{\alpha}_2 P_t + \hat{\alpha}_3 Y_t + e_t$

$\hat{\alpha}_2 = -.244$ (the negative sign is consistent with the law of demand)

$\hat{\alpha}_3 = .314$ (the positive sign is theoretically expected since food is a normal good)

PRF (supply function): $Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$

TSLS SRF: $Q_t = \hat{\beta}_1 + \hat{\beta}_2 P_t + \hat{\beta}_3 F_t + \hat{\beta}_4 A_t + e_t$

$\hat{\beta}_2 = .240$ (the positive sign is consistent with the law of supply)

$\hat{\beta}_3 = .256$ (the positive sign is theoretically expected since higher farming income from the preceding year should encourage farmers to increase supply)

$\hat{\beta}_4 = .253$ (the positive sign is theoretically expected since technological advances help increase supply)

Q26: Do the individual test of significance for the demand function. State your conclusions.

- The demand function: $Q_t = a_1 + a_2 P_t + a_3 Y_t + u_t$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	94.63330	7.920836	11.94739	0.0000
P	-0.243556	0.096484	-2.524313	0.0218
Y	0.313992	0.046944	6.688696	0.0000

The hypotheses associated with the individual tests of significance are:

$$\begin{cases} H_0: a_1 = 0 \\ H_1: a_1 \neq 0 \end{cases}$$

$$\begin{cases} H_0: a_2 = 0 \\ H_1: a_2 \neq 0 \end{cases}$$

$$\begin{cases} H_0: a_3 = 0 \\ H_1: a_3 \neq 0 \end{cases}$$

- From the Eviews output, the calculated absolute t-values for $H_0: a_1 = 0$, $H_0: a_2 = 0$, and $H_0: a_3 = 0$ are, respectively, 11.95, 2.52, and 6.69.
- The corresponding p -values are, respectively, .000, .0218, and .000. These p -values are all below .10, allowing us to reject $H_0: a_1 = 0$, $H_0: a_2 = 0$, and $H_0: a_3 = 0$.
- Rejection of the latter two null hypotheses allows us to conclude that the independent variables (P_t and Y_t) individually explain the behavior of the dependent variable (Q_t).

Q27: Do the individual test of significance for the supply function. State your conclusions.

- The supply function: $Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	49.53245	12.01052	4.124088	0.0008
P	0.240076	0.099934	2.402347	0.0288
F	0.255606	0.047250	5.409636	0.0001
A	0.252924	0.099655	2.537997	0.0219

The hypotheses associated with the individual tests of significance for monetary policy are:

$$\left\{ \begin{array}{l} H_0: \beta_1 = 0 \\ H_1: \beta_1 \neq 0 \end{array} \right\} \quad \left\{ \begin{array}{l} H_0: \beta_2 = 0 \\ H_1: \beta_2 \neq 0 \end{array} \right\} \quad \left\{ \begin{array}{l} H_0: \beta_3 = 0 \\ H_1: \beta_3 \neq 0 \end{array} \right\} \quad \left\{ \begin{array}{l} H_0: \beta_4 = 0 \\ H_1: \beta_4 \neq 0 \end{array} \right\}$$

- From the Eviews output, the calculated t-values for $H_0: \beta_1 = 0$, $H_0: \beta_2 = 0$, $H_0: \beta_3 = 0$, and $H_0: \beta_4 = 0$ are, respectively, 4.12, 2.40, 5.41, and 2.54.
- The corresponding p -values are, respectively, .0008, .0288, .0001, and .0219. These p -values are all below .10, allowing us to reject $H_0: \beta_1 = 0$, $H_0: \beta_2 = 0$, $H_0: \beta_3 = 0$, and $H_0: \beta_4 = 0$.
- Rejection of the latter three null hypotheses allows us to conclude that the independent variables (P_t , Y_t , and A_t) individually explain the behavior of the dependent variable (Q_t).

Q28: Do the joint test of significance (H_0 : population $R^2 = 0$) for the demand function. State your conclusion.

- The demand function: $Q_t = a_1 + a_2 P_t + a_3 Y_t + u_t$

Dependent Variable: Q
Method: Two-Stage Least Squares
Sample: 1921 1940
Included observations: 20
Instrument specification: C Y F A

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	94.63330	7.920836	11.94739	0.0000
P	-0.243556	0.096484	-2.524313	0.0218
Y	0.313992	0.046944	6.688696	0.0000
R-squared	0.754847	Mean dependent var	100.8982	
Adjusted R-squared	0.726005	S.D. dependent var	3.756497	
S.E. of regression	1.966320	Sum squared resid	65.72904	
F-statistic	23.80653	Durbin-Watson stat	2.009222	
Prob(F-statistic)	0.000012	Second-Stage SSR	84.02241	
J-statistic	2.535644	Instrument rank	4	
Prob(J-statistic)	0.111301			

- For the joint test of significance, the null and alternative hypotheses are:

$$\left\{ \begin{array}{ll} H_0: a_2 = a_3 = 0 & (H_0: \text{Population } R^2 = 0) \\ H_1: \text{At least one of these parameters} \neq 0 & (H_1: \text{Population } R^2 \neq 0) \end{array} \right.$$

- From the Eviews output, the calculated F-value is 23.81, with a p -value of $.000 < .10$. Accordingly, the estimated demand function is overall significant (or the population $R^2 \neq 0$).

Q29: Do the joint test of significance (H_0 : population $R^2 = 0$) for the supply function. State your conclusion.

- The supply function: $Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$

Dependent Variable: Q
Method: Two-Stage Least Squares
Sample: 1921 1940
Included observations: 20
Instrument specification: C F A Y

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	49.53245	12.01052	4.124088	0.0008
P	0.240076	0.099934	2.402347	0.0288
F	0.255606	0.047250	5.409636	0.0001
A	0.252924	0.099655	2.537997	0.0219
R-squared	0.639582	Mean dependent var	100.8982	
Adjusted R-squared	0.572004	S.D. dependent var	3.756497	
S.E. of regression	2.457555	Sum squared resid	96.63320	
F-statistic	10.70139	Durbin-Watson stat	2.384646	
Prob(F-statistic)	0.000420	Second-Stage SSR	74.21856	
J-statistic	0.000000	Instrument rank	4	

- For the overall test of significance, the null and alternative hypotheses are:

$$\begin{cases} H_0: \beta_2 = \beta_3 = \beta_4 = 0 & (H_0: \text{Population } R^2 = 0) \\ H_1: \text{At least one of these parameters} \neq 0 & (H_1: \text{Population } R^2 \neq 0) \end{cases}$$

- From the Eviews output, the calculated F-value is 10.70 with a p -value of $.0004 < .10$. Accordingly, the estimated supply function is overall significant (or the population $R^2 \neq 0$).

Q30: Is the quantity demanded or quantity supplied more sensitive to a change in price? Explain.

PRF (demand function):

$$Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 Y_t + u_t$$

TSLs SRF:

$$Q_t = \hat{\alpha}_1 + \hat{\alpha}_2 P_t + \hat{\alpha}_3 Y_t + e_t$$

$$\hat{\alpha}_2 = -.244 \quad (\text{the negative sign is consistent with the law of demand})$$

PRF (supply function):

$$Q_t = \beta_1 + \beta_2 P_t + \beta_3 F_t + \beta_4 A_t + v_t$$

TSLs SRF:

$$Q_t = \hat{\beta}_1 + \hat{\beta}_2 P_t + \hat{\beta}_3 F_t + \hat{\beta}_4 A_t + e_t$$

$$\hat{\beta}_2 = .240 \quad (\text{the positive sign is consistent with the law of supply})$$

- The answer to this question involves comparing $|\alpha_2|$ in the demand function with β_2 in the supply function.
- Note that α_2 measures the responsiveness of the quantity demanded to a change in price, and β_2 measures the responsiveness of the quantity supplied to a change in price.
- In this example, $|\hat{\alpha}_2| = .244$ and $\hat{\beta}_2 = .240$. With $.244 \approx .240$, we conclude that the quantity demanded and supplied are equally responsive to a change in price.

Q31: Are you satisfied with your estimates of the demand and supply functions? Explain.

The **Criteria for satisfactory estimates** are:

1. The estimated OLS SRL must pass the diagnostic tests. For example, we must have no specification error problem, no autocorrelation problem, no heteroscedasticity problem, and no multicollinearity problem. Also, the normality assumption should hold for C.I. estimation and hypothesis testing.
2. The parameter estimates should have the theoretically correct signs and sizes.
3. The slope parameter estimate(s) should be significantly different from zero.
4. The R^2 should be reasonably high (the population R^2 must be significantly different from zero).
5. The model must be stable in terms of parameters (no structural break).
6. The model must be dynamically stable.

Note that Criterion 6 is not relevant for this example. Why?